Relationship between permutation entropy and Kolmogorov-Sinai entropy for ergodic interval maps with positive entropy

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Permutation entropy is a measure for the complexity of dynamical systems. It is a question of interest for what classes of dynamical systems the permutation entropy is equal to the Kolmogorov-Sinai entropy, a classical measure for complexity.
It was shown in [1] by Unakofova, Unakafov and Keller that for one-dimensional systems the difference between the permutation and the Kolmogorov-Sinai entropy, which was considered using ordinal partitions, is related to the measure of specific sets V_d, $d = 2, 3, ...$, which depend on specific ordinal patterns. It was proven in [1] that for mixing interval maps the measure of V_d converges to zero for increasing d and that equality of permutation and K-S entropy depends on the rate of this convergence.
Here we show that for interval maps with strictly positive Kolmogorov-Sinai entropy, the weaker condition of ergodicity is sufficient for the measure of V_d to converge to zero. Proofing this we used tools from [1] together with the fact that ergodic maps exhibit Bernoulli-Factors with the same entropy to create upper bounds for the measure of V_d.

[1] V.A. Unakofova, A.M. Unakafov and K. Keller, *An approach to comparing Kolmogorov-Sinai and permutation entropy.* The European Physical Journal Special Topics, Vol. 222, No. 2, June 2013